Modification for French Rules Equations for Effective Length Factors of Steel Compression Members

Saeed Mazroa *

Magdy I. Salama **

* Assistant Prof., Faculty of Eng.&,Petroleum, Hadramout University, Yemen

** Lecturer, Faculty of Engineering, Kafrelsheikh University, Egypt

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Abstract:

In most current codes of design steel members and frames, specifications for the design of compression columns or of beam-column use the effective length factor; K. the effective length factor is employed to facilitate the design of framed members by transforming an end-restrained compressive member to an equivalent pinned-ended member. The effective length factor is obtained by solving the exact equations numerically which require many routine calculations or by using a pair of alignment charts for the two cases of braced frames and sway frames. The accuracy of these charts depends on the size of chart and the reader's sharpness of vision. Instead of using complicated equations or charts, simple equations are required to determine the effective length factor directly as a function of the rotational resistant at column ends (G_A, G_B). better equations have been available in the French design rules for steel structures since 1966, and have been included in the European recommendations of 1978. In this paper, modification to the French design rules equations for effective length factors are presented using multiple regressions for tabulated exact values corresponding to different practical values of the rotational resistance at column ends (G_A, G_B). The investigated equations are more accurate than current French rules equations that recommended in some steel constructions codes. Comparisons between the results of the present equations and those obtained by current equations with those obtained by exact solutions are given also in this paper.

KEYWORDS: Effective length; steel column; multiple regressions; new formula; braced frame; sway frame.

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الملخص :

في معظم المواصفات (الكود) لتصميم المنشــأت الحديديــةيعمل عامــل الطــول الفعال (k) على تسهيل تصميم عناصر الضغط ويتم الحصول عليه من خلال حل معادلات دقيقة والتي تتطلب العديد من العمليات الحسابية الروتينية أو باستخدام زوج من الرسوم البيانية و دقة هذه النتائج تعتمد على حجم الرسم البياني و دقة القارئ للرسومات البيانية.

بدلا من ذلك مطلوبة معادلات بسيطة لتحديد عامل الطول الفعال مباشرة لتحديدلمقاومة التناوبه على طرفي العمود (GA، GB) . وكانت أفضل المعــادلات المتاحــةفي المواصــفات الفرنسية لهياكل الصلب منذ عام ١٩٦٦ ، و أدرجت في التوصيات الأوروبية لعام ١٩٧٨ .

في هذه الورقة ، تم تعديل المعادلات الفرنسية لعوامل الطول الفعال باستخدام القيم العملية المختلفة من مقاومة التناوب على طرفي العمود (GA، GB) بالقيم المقابلة لها المحسوبه من الانحدارات المتعددة المجدولة الموافقه لها.وكانت النتائج أكثر دقة من القواعد الحالية للمعادلات الفرنسية التي أوصى بعض الكود الانشاءات الفولاذية .

تمت المقارنة في هذه الورقة ايضا" بين النتائج المحصلة من المعادلات الحالية مع تلك التي تم الحصول عليها عن طريق الحلول الدقيقة .

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(1) Introduction

The design of a column or of a beam-column starts with the evaluation of the elastic rotational resistance at both ends of the column (G_A, G_B), from which the effective length factor (K) is determined. The mathematically exact equations for braced and sway rigid frames were given by Barakat and chen [6]. These equations require many routine calculations, and it is well suited for tedious column and beam-column calculations. The other way to determine the effective length factor (K) is the using of a pair of alignment charts for braced frames and sway frames, which originally developed by O. J. Julian and L. S. Lawrence, and presented in detail by T. C. Kavanagh [8]. These charts are the graphic solutions of the mathematically exact equations and these are commonly used in most codes as manual of American institute of steel construction (LRFD and ASD) [1, 2] and the Egyptian code of practice for steel constructions (LRFD and ASD) [4, 5]. The accuracy of the alignment charts depends essentially on the size of the chart and on the reader's sharpness of vision. Also, having to read K-factors from an alignment chart in the middle of an electronic computation, in spreadsheet for instance prevents full automation and can be a source of errors.

Obviously, it would be convenient to have simple equations take the place of the charts which commonly used in most codes of steel constructions. The American Institute does publish equations but their lack of accuracy may be why they seem not to be used in steel design. Better equations have been available in the French design rule for steel structures since 1966, and have been included in the European recommendations of 1978 [3] are presented by Pierre Dumonteil [7].

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In this paper, modification for the French rules equations to get more accurate closed form equations for the determination of the effective length factors as a function of the rotational resistance at column ends. the presented equations are simple enough to be easily programmed within the confines of spreadsheet cell. For this reason, they may be useful to design engineers.

(2) Background to Exact and Approximate Equations

Consider a column *AB* elastically restrained at both ends. The rotational restraint at one end, *A* for instance, is presented by restraint factor G_A, expressing the relative stiffness of all the columns connected at *A* to that of all the beams framing into *A*:

$$G_A = \frac{\sum(I_C / L_C)}{\sum(I_b / L_b)} \qquad (1)$$

In the European Recommendation, another two factors β_A and β_B are used (rather than G_A and G_B as in French Rules). The definition of β differs from that of *G*, since, at each column end:

$$\beta = \frac{\sum(I_b / L_b)}{\sum(I_b / L_b) + \sum(I_C / L_C)} \qquad (2)$$

The mathematical relation between G and β is simple:

$$\beta = 1/(1+G) \qquad (3)$$

Europeans tend to prefer β to *G* because a hinge means β =0 and fixity β =1. Obviously, the *K*-factor will be the same if the same elements are introduced in *G* and β.

(2.1) Braced Frames

Braced frames are frames in which the side sway is effectively prevented as shown in Figure (1-a), and, therefore, the *K*-factor is never greater than 1.0. The side sway prevented alignment chart is the graphic solution of the following mathematical equation:

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$$\frac{G_A G_B}{4}(\pi / K)^2+\left(\frac{G_A+G_B}{2}\right)\left(1-\frac{\pi / K}{\tan(\pi / K)}\right)+2\frac{\tan(\pi / 2K)}{\pi / K}=1 \quad (4)$$

This equation is mathematically exact, in that certain physical assumptions are exactly translated in mathematical terms. Whether these assumptions can be reasonably extended to a specific structure is a matter for the designer to decide.

For the transcendental Eq. 4, which can only be solved by numerical methods, the French Rules propose the following approximate solution:

$$K=\frac{3\,G_A G_B+1.4(G_A+G_B)+0.64}{3\,G_A G_B+2.0(G_A+G_B)+1.28} \quad (5)$$

(2.2) Sway Frames

If a rigid frame depends solely on frame action to resist lateral forces, its side sway is permitted as shown in Figure (1-b). In this case, the *K*-factor is never smaller than 1.0. The mathematical equation for the permitted sway case is:

$$\frac{G_A G_B(\pi / K)^2-36}{6(G_A+G_B)}=\frac{\pi / K}{\tan(\pi / K)} \quad (6)$$

Although simpler than Eq. 4, this equation cannot be solved in closed form either. The French Rules recommend the following approximate solution:

$$K=\sqrt{\frac{1.6\,G_A G_B+4.0(G_A+G_B)+7.5}{G_A+G_B+7.5}} \quad (7)$$

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(3) Theoretical Formulas

In this section, Firstly, forms of French rules equations are assumed as follows

for braced frames,

$$K=\frac{a\ G_A\ G_B+b\ (G_A+G_B)+c}{d\ G_A\ G_B+e\ (G_A+G_B)+f} \qquad (8)$$

for sway frames,

$$K=\left(\frac{g\ G_A\ G_B+h\ (G_A+G_B)+i}{G_A+G_B+j}\right)^k \qquad (9)$$

The parameters (*a, b, c ...k*) are obtained by applying multiple regressions analyses using the following procedure

- 300 pairs of different practical values of the rotational resistance at column ends (G_A, G_B) are selected and the corresponding K values for sway and braced framework are used to form assumed equations, which can be tested.
- The exact values of K factor were obtained by trial and error applying equations (Eqs. 4, 6) which solved numerically for both prevented and permitted sway frames. The resulting values are approximated to the nearest higher integer numbers.
- Using computer software, multiple regressions analyses based on the least-squared method are developed for each suggested formula to obtain the parameters which give the least standard error and less than that corresponding to the current French Rules equations.

Then, very accurate and simple equations are investigated to determine the effective length factor K for braced and sway steel frames as described in the following.

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In case of braced frames; the equation of effective length factor *K* is

$$K=\frac{3\,G_A\,G_B+1.4(G_A+G_B)+0.695}{3\,G_A\,G_B+2(G_A+G_B)+1.39} \qquad (10)$$

Where $0 \le G_A \le 100, \quad 0 \le G_B \le 100$

In case of sway frames; to get good results, two equations of effective length factor are investigated according to the domain of the rotational resistance at column ends (G_A, G_B) as follows

$$K=\left(\frac{0.97\,G_A\,G_B+3.3(G_A+G_B)+6.7}{G_A+G_B+6.9}\right)^{0.6} \qquad (11)$$

where $0 \le G_A \le 10, \quad 0 \le G_B \le 10$

and,

$$K=\left(\frac{1.4\,G_A\,G_B+3.7(G_A+G_B)+6.15}{G_A+G_B+6.45}\right)^{0.52} \qquad (12)$$

Where $10 \prec G_A \le 100$ or $10 \prec G_B \le 100$

(4) Accuracy of present equations

The accuracy that we can readily measure is of course the mathematical accuracy, that is, the comparison of the results given by the obtained formulas to those obtained by solving the corresponding exact equations. First, take a look to the accuracy of the most common alignment charts and the French Rules equations.

The accuracy of the alignment charts depends essentially on the size of the charts and the reader's sharpness of vision. This accuracy may be about five percent in small charts. In the other hand, the French Rules [7] indicate that Eq. (5), used for braced frames, has an accuracy of -0.50 percent to +1.50 percent while Eq. (7) used for sway frames, is accurate within two percent.

The percentage of errors for all points considered in the present regressions analyses (about 300 point for each case) indicates that

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the investigated equations in the present work is accurate within 1.0 percent for both sway and braced frames.

(5) Comparison of the results

Using a few sample points, tables (1, 2) show the comparison of the effective length factor K obtained by Equations [8 – 10] of the present work (P.W.), and that obtained by the current French Rules equations [5, 7] with exact values for braced frames and sway frames respectively.

It can be noticed that although the present equations are simple, they gave results very close to the exact values and more accurate comparing with the solution by the current French Rules equations. Then, the present equations can be rather used by the designer engineers with sufficient confidence.

Also, as shown in Table (3), the standard error's of the obtained formula Eq. (8) is about two-third of that of French Rules Eq. (5) in case of braced frames while in the case of sway frames the standard error of Eqs. (9, 10) has less than one-half of that of French Rules Eq. (7).

(6) Conclusions

In this paper, new simple closed form equations (modified French rules equations) for the determination of the effective length factor K of steel columns are investigated by multiple regressions analyses using the results of the exact solution. The analysis is carried out in a wide range of the rotational resistance at column ends G_A, G_B (from 0 to 100).

The obtained equations are simple and more accurate than the current equations and they may be simple for design purposes. Their simple closed forms make them easily programed within the confines of spreadsheet cell and generally well suited for computer use.

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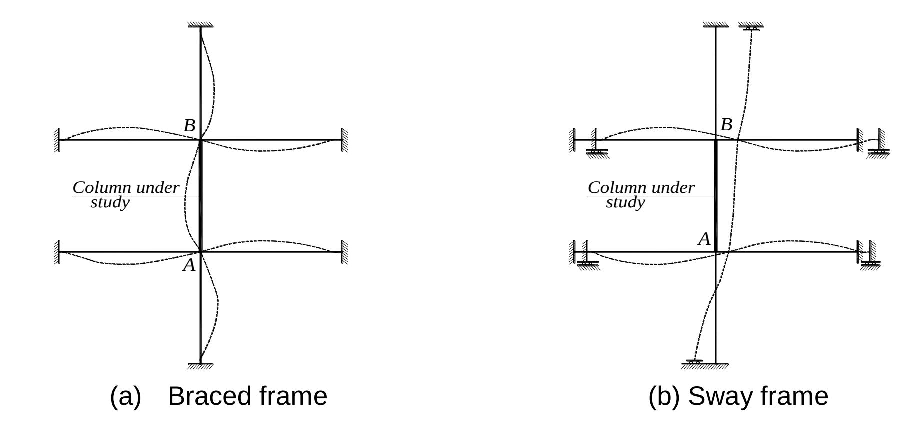

Fig. 1 Braced and sway frames

Table 1. Comparison of K-factors obtained by P. W., and French Rules with exact results [side sway is prevented]

G_A	G_B	exact value	P. W.	% Diff.	French Rule [7]	% Diff.
0.1	0.4	0.603	0.604	0.10	0.608	0.88
0.25	0.25	0.611	0.614	0.47	0.619	1.30
0.1	0.9	0.648	0.646	-0.28	0.651	0.42
0.25	0.75	0.672	0.672	0.05	0.677	0.79
0.5	0.5	0.686	0.687	0.17	0.692	0.92
0.1	1.9	0.683	0.682	-0.14	0.685	0.36
0.25	1.75	0.716	0.717	0.18	0.721	0.68
0.5	1.5	0.751	0.752	0.13	0.756	0.62
1	1	0.774	0.774	0.02	0.778	0.49
0.5	4.5	0.792	0.796	0.54	0.798	0.77
1	4	0.840	0.842	0.24	0.844	0.43
2.5	2.5	0.877	0.877	0.05	0.879	0.20
0.5	9.5	0.806	0.812	0.71	0.813	0.88
1	9	0.858	0.862	0.42	0.862	0.52
2.5	7.5	0.913	0.914	0.08	0.914	0.15
5	5	0.930	0.931	0.06	0.931	0.11
50	4	0.952	0.953	0.10	0.953	0.11
50	10	0.977	0.977	0.04	0.977	0.04
100	50	0.994	0.994	0.01	0.994	0.01

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Table 2. Comparison of K-factors obtained by P. W., and French Rules with exact results [side sway is permitted]

G_A	G_B	exact value	P. W.	% Diff.	French Rule [7]	% Diff.
0.1	0.4	1.083	1.078	-0.45	1.093	0.96
0.25	0.25	1.083	1.080	-0.29	1.095	1.15
0.1	0.9	1.159	1.158	-0.09	1.170	0.99
0.25	0.75	1.162	1.164	0.21	1.178	1.40
0.5	0.5	1.164	1.169	0.40	1.183	1.65
0.1	1.9	1.286	1.283	-0.22	1.290	0.30
0.25	1.75	1.295	1.297	0.14	1.306	0.84
0.5	1.5	1.307	1.314	0.53	1.326	1.44
1	1	1.317	1.327	0.77	1.342	1.87
0.5	4.5	1.575	1.575	0.03	1.577	0.15
1	4	1.634	1.638	0.23	1.647	0.78
2.5	2.5	1.711	1.716	0.28	1.732	1.23
0.5	9.5	1.777	1.783	0.34	1.774	-0.15
1	9	1.874	1.881	0.36	1.881	0.36
2.5	7.5	2.092	2.093	0.06	2.104	0.59
5	5	2.228	2.222	-0.26	2.236	0.36
50	4	2.949	2.956	0.24	2.973	0.81
50	10	3.948	3.940	-0.21	3.939	-0.22
100	50	7.476	7.513	0.49	7.393	-1.12

Table 3. Comparison of standard error of the obtained equations and French Rules

	Equations		s_e
braced frames	French rule equation	(5)	0.0048
	modified equation	(8)	0.0033
sway frames	French rule equation	(7)	0.0212
	modified equation	(9)	0.0043
		(10)	0.0137

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