Generating Functions For Quadruple Hyper geometric Function

اشتقاق بعض من الدوال المولدة للدوال الفوق هندسية الرباعية

Ahmed Ali Atash*

Ala'a Mohammed Obad**

* *Department of Mathematics, Faculty of Education-Shabowh, Aden University*

** *Department of Mathematics, Faculty of Education-Aden, Aden University*

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Generating Functions For Quadruple Hypergeometric Functions

Abstract *:*

The aim of this paper is to derive some of *Generating Functions* for the quadruple *Hypergeometric* functions $(K_3, K_{10}, K_{13}, K_{16}$ *and* $K_{18})$. The results are derived with the help of Laplace integral. A number of *Generating Functions* of such other types of hypergeometric functions are also derived as special cases of our main results.

Key words: Quadruple hypergeometric functions, *generatingfunctions*, Laplace integral,hypergeometric functions

الملخص :

هدف بحثنا هذا اشتقاق بعض من الدوال المولدة للدوال الفوق هندسية الرباعية (K_{18} ,K_{16} ,K_{13} ,K_{10} ,K_3) تم الحصول على هذه النتائج بمساعدة تكاملات لابلاس، أيضًا تم عرض العديد من المولدات لأنواع أخرى من الدوال الفوق هندسية وذلك كحالات خاصة لنتائج البحث الرئيسة .

الكلمات المفتاحية : الدوال الفوق هندسية الرباعية ، الدوال المولدة ، تكامل لابلاس، الدوال الفوق هندسية .

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1. Introduction :

The following are the definitions and the Laplace integral representations of the quadruple hypergeometric functions K_i $(i=3,10,13,16$ *and* $18)$ *[1;p,78-83] :*

$$K_3\left(a,a,a,a;b_1,b_1,b_2,b_2;c_1,c_2,c_2,c_1;x,y,z,t\right)$$

$$=\sum_{p,q,r,s=0}^{\infty}\frac{(a)_{p+q+r+s}(b_1)_{p+q}(b_2)_{r+s}\,x^p y^q z^r t^s}{(c_1)_{p+s}(c_2)_{q+r}\,p!q!r!s!}\qquad(1.1)$$

$$=\frac{1}{\Gamma(b_1)\Gamma(b_2)}\int_0^{\infty}\int_0^{\infty}e^{-u-v}u^{b_1-1}v^{b_2-1}\Psi_2(a;c_1,c_2;xu+tv,yu+zv)\,du\,dv\quad,(1.2)$$

$$K_{10}(a,a,a,a;b,b,c_1,c_2;d_1,d_2,d_3,d_4;x,y,z,t)$$

$$=\sum_{m,n,p,q=0}^{\infty}\frac{(a)_{m+n+p+q}(b)_{m+n}(c_1)_p(c_2)_q\,x^m y^n z^p t^q}{(d_1)_m(d_2)_n(d_3)_p(d_4)_q\,m!n!p!q!}\qquad(1.3)$$

$$=\frac{1}{\Gamma(a)}\int_0^{\infty}e^{-u}u^{a-1}\,\Psi_2(b;d_1,d_2;xu,yu)\,{}_1F_1(c_1;d_3;zu)\,{}_1F_1(c_2;d_4;tu)\,du\qquad(1.4)$$

$$K_{13}(a,a,a,a;b_1,b_2,b_3,b_4;c,c,d_1,d_2;x,y,z,t)$$

$$=\sum_{m,n,p,q=0}^{\infty}\frac{(a)_{m+n+p+q}(b_1)_m(b_2)_n(b_3)_p(b_4)_q\,x^m y^n z^p t^q}{(c)_{m+n}(d_1)_p(d_2)_q\,m!n!p!q!}\qquad(1.5)$$

$$=\frac{1}{\Gamma(a)}\int_0^{\infty}e^{-u}u^{a-1}\,\Phi_2(b_1,b_2;c;xu,yu)\,{}_1F_1(b_3;d_1;zu)\,{}_1F_1(b_4;d_2;tu)\,du\,(1.6)$$

$$K_{16}\left(a_1,a_2,a_3,a_4;b;x,y,z,t\right)$$

$$=\sum_{m,n,p,q=0}^{\infty}\frac{(a_1)_{m+n}(a_2)_{m+p}(a_3)_{n+q}(a_4)_{p+q}\,x^m y^n z^p t^q}{(b)_{m+n+p+q}\,m!n!p!q!}\qquad(1.7)$$

$$=\frac{1}{\Gamma(a_2)\Gamma(a_3)}\int_0^{\infty}\int_0^{\infty}e^{-u-v}u^{a_2-1}v^{a_3-1}\Phi_2(a_1,a_4;b;xu+yv,zu+tv)\,du\,dv$$

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$$. (1.8)\ K_{18}\left(a_1, a_2, a_3, b_1, b_2; c; x, y, z, t\right)$$

$$= \sum_{m,n,p,q=0}^{\infty} \frac{(a_1)_{m+n}(a_2)_{m+q}(a_3)_{n+p}(b_1)_p(b_2)_q x^m y^n z^p t^q}{(c)_{m+n+p+q}\ m!n!p!q!} \quad (1.9)$$

$$= \frac{1}{\Gamma(a_1)\Gamma(b_1)\Gamma(b_2)} \int_0^{\infty}\int_0^{\infty}\int_0^{\infty} e^{-u-v-w} u^{a_1-1} v^{b_1-1} w^{b_2-1}$$

$$\Phi_2(a_2, a_3; c; xu+tw, yu+zv)\, du\, dv dw\ . \quad (1.10)$$

2. Generating functions

In this section we have established the following Generating relations :

$$\sum_{n=0}^{\infty} \frac{(b_1)_n (b_2)_n w^n}{(d)_n n!} K_3$$

$$\left(a,a,a,a; b_1+n, b_1+n, b_2+n, b_2+n; c_1, c_2, c_2, c_1; x,x,z,z\right)$$

$$= F^{(3)}\left[\begin{matrix} -:: & a, \tfrac{1}{2}(c_1+c_2), \tfrac{1}{2}(c_1+c_2-1) & ; b_2 ; b_1 : -; -; -; \\ - :: & c_1, c_2, c_1+c_2-1 & ; - ; - : -; -; d; \end{matrix} 4x, 4z, w\right], (2.1)$$

$$\sum_{n=0}^{\infty} \frac{(a)_n w^n}{n!} K_3\left(a+n, a+n, a+n, a+n; b_1, b_1, b_2, b_2; c_1, c_2, c_2, c_1; x, x, y, y\right)$$

$$= F^{(3)}\left[\begin{matrix} a :: & \tfrac{1}{2}(c_1+c_2), \tfrac{1}{2}(c_1+c_2-1) & ; -; - : b_1; b_2; -; \\ - :: & c_1, c_2, c_1+c_2-1 & ; -; - : -; -; -; \end{matrix} 4x, 4y, w\right], (2.2)$$

$$\sum_{n=0}^{\infty} \frac{(a_2)_n (a_3)_n w^n}{n!} K_{16}\left(a_1, a_2+n, a_3+n, b_1-a_1; b_1; x, y, z, t\right)$$

$$= (1-z)^{-a_2}(1-t)^{-a_3}\ F^{(3)}\left[\begin{matrix} - :: a_1 ; a_3 ; a_2 : -; -; -; \\ - :: b_1 ; - ; - : -; -; -; \end{matrix} \frac{x-z}{1-z}, \frac{y-t}{1-t}, \frac{w}{(1-z)(1-t)}\right], (2.3)$$

$$\sum_{n=0}^{\infty} \frac{(a_2)_n (a_3)_n w^n}{(b_2)_n n!} K_{16}\left(a_1, a_2+n, a_3+n, b_1-a_1; b_1; x, y, z, t\right)$$

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$$\left[= (1-z)^{-a_2}(1-t)^{-a_3} H_A\left[a_3, a_2, a_1; b_2, b_1 ;, \frac{w}{(1-z)(1-t)}, \frac{x-z}{1-z}, \frac{y-t}{1-t}\right]\right], (2.4)$$

$$\sum_{n=0}^{\infty} \frac{(a_1)_n (a_2)_n w^n}{(b_1)_n n!} K_{16}\left(a, a_1+n, a_2+n, a_3; b_1+n ; x, y, x, y\right)$$

$$= H_C\left[a_1, a+a_3, a_2; b_1 ; x, y, w\right], (2.5)$$

$$\sum_{n=0}^{\infty} \frac{(b_1)_n (b_2)_n w^n}{n!} K_{18}\left(a_1+n, a_2, c_1-a_2, b_1+n, b_2+n; c_1; x, x, z, t\right)$$

$$= (1-x)^{-a_1}(1-z)^{-b_1} F^{(3)}\left[\begin{matrix} - :: a_2 ; b_1; b_2 :-;-;-; \\ - :: c_1 ;-; - :-;-;-; \end{matrix} t, \frac{-z}{1-z}, \frac{w}{(1-x)(1-z)}\right], (2.6)$$

$$\sum_{n=0}^{\infty} \frac{(b_1)_n (b_2)_n w^n}{(c_2)_n n!} K_{18}\left(a_1+n, a_2, c_1-a_2, b_1+n, b_2+n; c_1; x, x, z, t\right)$$

$$= (1-x)^{-a_1}(1-z)^{-b_1} H_A\left[b_1, b_2, a_2 ; c_2, c_1 ; \frac{w}{(1-x)(1-z)}, t, \frac{-z}{1-z}\right], (2.7)$$

$$\sum_{n=0}^{\infty} \frac{(b_1)_n (b_2)_n w^n}{(c_1)_n n!} K_{18}\left(a_1+n, a_2, c_1-a_2+n, b_1+n, b_2+n; c_1+n; x, x, z, t\right)$$

$$= (1-x)^{-a_1}(1-z)^{-b_1} H_C\left[b_2, a_2, b_1 ; c_1 ; t, \frac{z}{z-1}, \frac{w}{(1-x)(1-z)}\right], (2.8)$$

$$\sum_{n=0}^{\infty} \frac{(a)_n (b)_n w^n}{(c)_n n!} K_{10}\left(a+n, a+n, a+n, a+n; b_1, b_1, c_1, c_2; b_1, b_1, 2c_1, 2c_2; x, y, 2z, 2t\right)$$

$$= A^{-a}\ {}^{(3)}H_4^{(4)}\left[a, b ; b_1, c_1+\frac{1}{2}, c_2+\frac{1}{2}, c ; \frac{xy}{A^2}, \frac{z^2}{4A^2}, \frac{t^2}{4A^2}, \frac{w}{A}\right], (2.9)$$

$$\sum_{n=0}^{\infty} \frac{(a/2)_n ((a+1)/2)_n w^n}{(d)_n n!}$$

$$K_{10}\left(a+2n, a+2n, a+2n, a+2n; b_1, b_1, c_1, c_2; b_1, b_1, 2c_1, 2c_2; x, y, 2z, 2t\right)$$

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$$= A^{-a}\, F_C^{(4)}\left[\frac{a}{2},\frac{a+1}{2};b_1,c_1+\frac{1}{2},c_2+\frac{1}{2},d\ ;\ \frac{4xy}{A^2}\ ,\ \frac{z^2}{A^2},\frac{t^2}{A^2},\frac{w}{A^2}\right], (2.10)$$

$$\sum_{n=0}^{\infty}\frac{(a)_n(b)_n w^n}{(c)_n n!}K_{13}\left(a+n,a+n,a+n,a+n;b_1,b_1,c_1,c_2;2b_1,2b_1,2c_1,2c_2;2x,2y,2z,2t\right)$$

$$= A^{-a}\ {}^{(3)}H_4^{(4)}\left[a,b\ ;b_1+\frac{1}{2},c_1+\frac{1}{2},c_2+\frac{1}{2},c\ ;\ \frac{(x-y)^2}{4A^2}\ ,\ \frac{z^2}{4A^2},\frac{t^2}{4A^2},\frac{w}{A}\right](2.11)$$

and

$$\sum_{n=0}^{\infty}\frac{(a/2)_n((a+1)/2)_n w^n}{(d)_n n!}$$

$$K_{13}\left(a+2n,a+2n,a+2n,a+2n;b_1,b_1,c_1,c_2;2b_1,2b_1,2c_1,2c_2;2x,2y,2z,2t\right)$$

$$= A^{-a}\, F_C^{(4)}\left[\frac{a}{2},\frac{a+1}{2};b_1+\frac{1}{2},c_1+\frac{1}{2},c_2+\frac{1}{2},d\ ;\frac{(x-y)^2}{A^2}\ ,\frac{z^2}{A^2},\frac{t^2}{A^2},\frac{w}{A^2}\right],(2.12)$$

where $F^{(3)}$ is the General Triple *Hypergeometric* Series *[3; p.44]*, H_A and H_C are the *Srivastava's triple* series [3;p.43] , $F_C^{(4)}$ is the *Lauricella's Function* of four variables [3;p.33] , ${}^{(3)}H_4^{(4)}$ is the *Generalized Horn's Function* [1;p,97] and $A=(1-x-y-z-t)$.

3. Results Required

The following results will be required in our present investigations (c.f.[2] and [3]) :

$$\Psi_2\left[a;c,c';x,x\right]={}_3F_3\left[a,\frac{c+c'}{2},\frac{c+c'-1}{2};c,c',c+c'-1;\ 4x\right](3.1)$$

$$F\begin{matrix}p:0;0\\q:0;0\end{matrix}\left[\begin{matrix}(a_P):\ -;-\ ;\\(b_q):\ -;-\ ;\end{matrix}x,y\right]={}_PF_q\left[\begin{matrix}(a_P);\\(b_q);\end{matrix}x+y\right](3.2)$$

$$\Psi_2\left[a;a,a;x,y\right]=e^{x+y}\,{}_0F_1\left[-\quad;\quad a;xy\right](3.3)$$

$$\Phi_2\left[a,b-a\ ;b\ ;\ x,y\ \right]=e^{y}\,{}_1F_1\left[a\ ;\ b\ ;x-y\right]\ (3.4)$$

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$$ {}_1F_1\left[\begin{matrix} a \\ 2a \end{matrix} ; x\right] = e^{\frac{x}{2}} {}_0F_1\left[-; a+\frac{1}{2}; \frac{x^2}{16}\right] \quad (3.5) $$

$$ \int_0^\infty e^{-su} u^{a-1}\, du = \frac{\Gamma(a)}{s^a} \qquad (3.6) $$

$$ \int_0^\infty e^{-su} u^{a-1} {}_0F_1(-; d_1; xu^2) {}_0F_1(-; d_2; yu^2)\, du $$

$$ = \frac{\Gamma(a)}{s^a} F_4\left[\frac{a}{2}, \frac{a+1}{2}; d_1, d_2; \frac{4x}{s^2}, \frac{4y}{s^2}\right] (3.7) $$

$$ (\lambda)_{2n} = 2^{2n} \left(\tfrac{1}{2}\lambda\right)_n \left(\tfrac{1}{2}\lambda + \tfrac{1}{2}\right)_n , \qquad n = 0,1,2,\cdots\cdots \qquad (3.8) $$

where the *Functions* ${}_pF_q$ is the *Generalized Hypergeometric Function*, F_4 is *Appell's Function* , Φ_2 and Ψ_2 are the *confluent Hypergeometric Function* of two variables and $F^{\,p:q;k}_{\,l:m;n}$ the *Kampé de Fériet Function* of two variables (c.f.[4]) .

4. Proof of the results :

To prove (2.1), we proceed as follows :

Let us denote the left hand side of (2.1) by I and using (1.2) , we get

$$ I = \sum_{n=0}^{\infty} \frac{(b_1)_n (b_2)_n w^n}{(d)_n n!} $$

$$ \frac{1}{\Gamma(b_1+n)\Gamma(b_2+n)} \int_0^\infty \int_0^\infty e^{-u-v} u^{b_1+n-1} v^{b_2+n-1} \Psi_2(a; c_1, c_2; xu+zv, xu+zv)\, du\, dv $$

Now, using (3.1) and (3.2) ,we get

$$ I = \sum_{n=0}^{\infty} \frac{(b_1)_n (b_2)_n w^n}{(d)_n \Gamma(b_1+n)\Gamma(b_2+n) n!} $$

$$ \int_0^\infty \int_0^\infty e^{-u-v} u^{b_1+n-1} v^{b_2+n-1} $$

$$ F\begin{matrix} 3:0;0 \\ 3:0;0 \end{matrix}\left[\begin{matrix} a\ , \frac{1}{2}(c_1+c_2)\ , \frac{1}{2}(c_1+c_2-1): & -;-\ ; \\ c_1,\ c_2\ ,\ c_1+c_2-1 & : -;-\ ; \end{matrix} 4xu, 4yv\right] du\, dv $$

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Expressing the Kampé de Fériet function as double series and using (3.6), we have

$$I=\sum_{n=0}^{\infty}\frac{(b_1)_n(b_2)_n\,w^n}{(d)_n\Gamma(b_1+n)\Gamma(b_2+n)n!}$$

$$\sum_{r=0}^{\infty}\sum_{s=0}^{\infty}\frac{(a)_{r+s}\,(\tfrac{1}{2}(c_1+c_2))_{r+s}(\tfrac{1}{2}(c_1+c_2-1))_{r+s}(4x)^r(4z)^s\Gamma(b_1+n+r)\Gamma(b_2+n+s)}{r!\;s!\;(c_1)_{r+s}\,(c_2)_{r+s}\,(c_1+c_2-1)_{r+s}}$$

$$I=F^{(3)}\left[\begin{matrix}-::a,\tfrac{1}{2}(c_1+c_2),\tfrac{1}{2}(c_1+c_2-1)\;;b_2\,;\;b_1:-;-;-;\\ -::\quad c_1,\;c_2\,,\;c_1+c_2-1\quad;-\,;\;-:-;-;d\,;\end{matrix}4x,4z,w\right]$$

This completes the proof of (2.1),the proofs of (2.2)-(2.8) are similarly.

To prove (2.9), we proceed as follows :

Let us denote the left hand side of (2.9) by I, using (1.4) ,(3.3) and (3.5) , we get

$$I=\sum_{n=0}^{\infty}\frac{(a)_n(b)_n\,w^n}{(c)_n n!\;\Gamma(a+n)}$$

$$\int_0^{\infty}e^{-(1-x-y-z-t)u}\,u^{a+n-1}\;{}_0F_1(-;b_1;x\,y\,u^2)\;{}_0F_1(-;c_1+\frac{1}{2};\frac{z^2}{16}u^2)\;{}_0F_1(-;c_2+\frac{1}{2};\frac{t^2}{16}u^2)du$$

Now, expressing the first ${}_0F_1$ into power series and using (3.7),we get

$$I=\sum_{n,m=0}^{\infty}\frac{(a)_n(b)_n\,w^n(xy)^m\Gamma(a+n+2m)}{(c)_n(b_1)_m n!m!\;\Gamma(a+n)\,A^{a+n+2m}}$$

$$F_4\left[\frac{a+n+2m}{2},\frac{a+n+2m+1}{2};c_1+\frac{1}{2},c_2+\frac{1}{2};\frac{z^2}{A^2},\frac{t^2}{A^2}\right].$$

Expressing *Appell's Function* F_4 as double series and using (3.8),we will gettheright hand side of (2.9),which complete the proof of (2.9). The proofs of (2.10) - (2.12) are similarly.

-ISSN : 2410-7727
-ISSN : 2410-7735

5. Particular Cases :

In this section ,we shall mention some interesting generating *relations*as particular cases of our main results.

In (2.1), if we put $a=c_1=c_2$, then we get

$$\sum_{n=0}^{\infty}\frac{(b_1)_n(b_2)_n w^n}{(d)_n n!}K_3\left(a,a,a,a;b_1+n,b_1+n,b_2+n,b_2+n;a,a,a,a\,;x,x,z,z\right)$$
$$=H_A\left(b_2,b_1,a-\frac{1}{2};d,\,2a-1\,;w\,,4x,\,4z\right)\quad,(5.1)$$

On taking $t=0$ in (2.3) , (2.4) , (2.7) and (2.8) respectively ,we get the following results :

$$\sum_{n=0}^{\infty}\frac{(a_2)_n(a_3)_n w^n}{n!}F_T\left(b_1-a_1,a_1\ a_1\,;a_2+n\ ,a_3+n\ ,a_2+n;b_1,b_1,b_1;\ z,y,x\right)$$
$$=(1-z)^{-a_2}\ F^{(3)}\left[\begin{matrix}-::\ a_1\ ;a_3;\ a_2\ :-;-;-; \\ -::\ b_1\ ;-;\ -:-;-;-;\end{matrix}\frac{x-z}{1-z},y,\frac{w}{(1-z)}\right]\ ,\qquad(5.2)$$

$$\sum_{n=0}^{\infty}\frac{(a_2)_n(a_3)_n w^n}{(b_2)_n n!}F_T\left(b_1-a_1,a_1\ a_1\,;a_2+n\ ,a_3+n\ ,a_2+n;b_1,b_1,b_1;\ z,y,x\right)$$
$$=(1-z)^{-a_2}\ H_A\left[\ a_3,a_2,a_1;b_2,b_1\,;\frac{w}{(1-z)},\frac{x-z}{1-z},y\right],\ (5.3)$$

$$\sum_{n=0}^{\infty}\frac{(b_1)_n(b_2)_n w^n}{(c_2)_n n!}F_T\left(a_2,c_1-a_2,c_1-a_2,a_1+n,b_1+n,a_1+n;c_1,c_1,c_1;\ x,z,x\right)$$
$$=(1-x)^{-a_1}(1-z)^{-b_1}\ F_2\left[\ b_1,a_2,b_2;c_1,c_2;\frac{-z}{1-z},\frac{w}{(1-x)(1-z)}\right]\qquad(5.4)$$

and

$$\sum_{n=0}^{\infty}\frac{(b_1)_n(b_2)_n w^n}{(c_1)_n n!}F_T\left(b_1+n,a_1+n,a_1+n,c_1-a_2+n,a_2,c_1-a_2+n\,;c_1+n,c_1+n,c_1+n;\ z,x,x\right)$$

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$$= (1-x)^{-a_1}(1-z)^{-b_1}\; F_1\left[b_1, a_2, b_2\; ; c_1\; ; \frac{z}{z-1}, \frac{w}{(1-x)(1-z)}\right] \quad (5.5)$$

respectively , where F_1 and F_2 are *Appell's Functions* [4;p.53] and F_T is *Saran's Function*[4;p. 67] .

In (2.6), if we put $b = c \; and \; x = y = \frac{1}{2}v$, then we get

$$\sum_{n=0}^{\infty} \frac{(a)_n w^n}{n!} K_{10}\left(a+n, a+n, a+n, a+n; b_1, b_1, c_1, c_2; b_1, b_1, 2c_1, 2c_2; \frac{1}{2}v, \frac{1}{2}v, 2z, 2t\right)$$

$$= K^{-a}\, F_C^{(3)}\left[\frac{a}{2}, \frac{a+1}{2}; b_1, c_1 + \frac{1}{2}, c_2 + \frac{1}{2}\; ; \frac{v^2}{K^2}, \frac{z^2}{K^2}, \frac{t^2}{K^2}\right] \quad (5.6)$$

In (2.7), if we put $b = c$, then we get

$$\sum_{n=0}^{\infty} \frac{(a)_n w^n}{n!} K_{13}\left(a+n, a+n, a+n, a+n; b_1, b_1, c_1, c_2; 2b_1, 2b_1, 2c_1, 2c_2; 2x, 2y, 2z, 2t\right)$$

$$= K^{-a}\, F_C^{(3)}\left[\frac{a}{2}, \frac{a+1}{2}; b_1 + \frac{1}{2}, c_1 + \frac{1}{2}, c_2 + \frac{1}{2}\; ; \frac{(x-y)^2}{K^2}, \frac{z^2}{K^2}, \frac{t^2}{K^2}\right], \quad (5.7)$$

where $F_C^{(3)}$ the *Lauricella's Function* of three variables [4; p.60] and $K = (1-x-y-z-t-w)$.

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